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ON CRACK PROPAGATION IN SOLIDS†

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Abstract—The author's recent investigations in the crack propagation theory are summarized in this paper.

1. THE MODIFIED CONCEPT OF GRIFFITH

GRIFFITH introduced into fracture mechanics the idea of the free surface energy of elastic bodies. We consider the application of this idea to more complicated rheological models within the framework of continuum mechanics.

A. *The general governing equation*

Let the deformed body have opening mode cracks. We shall regard cracks as mathematical cuts, hence we neglect the finiteness of deformations which is always present at the tip of a real crack. Consider the fine structure in the vicinity of an arbitrary point on the contour of the crack (the linear dimension of the vicinity under study is assumed to be infinitesimal; in fact, however, it is large compared to the radius of the crack tip). If we confine ourselves to processes with contribution by mechanical and heat energy only, then the energy equation of fine structure can be reduced, in the case of the growing crack, to the following equivalent forms:

$$\lim_{R\to 0}\left\{R\int_0^{2\pi}[(Э+K)\cos\theta-A]|_{x^2+y^2=R^2}\,\mathrm{d}\theta\right\}=2\gamma \tag{1.1}$$

or

$$\lim_{R\to 0,\delta/R\to 0}\left\{\int_{-R}^{+R}\left(\sigma_y\frac{\partial v}{\partial x}\right)\bigg|_{y=\delta}\mathrm{d}x\right\}=\gamma \tag{1.2}$$

Here

$$Э=\int\sigma_x\,\mathrm{d}\varepsilon_x+\sigma_y\,\mathrm{d}\varepsilon_y+2\tau_{xy}\,\mathrm{d}\varepsilon_{xy}$$

$$K=\frac{1}{2}\rho l^2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2\right]$$

$$A=(\sigma_x\cos\theta+\tau_{xy}\sin\theta)\frac{\partial u}{\partial x}+(\tau_{xy}\cos\theta+\sigma_y\sin\theta)\frac{\partial v}{\partial x}$$

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ρ is the mass density, $\dot{l}$ is the crack extension velocity at the point O, γ is the specific free surface energy (the true surface energy). The remaining symbols are illustrated in Fig. 1.

The governing equation (1.1) or (1.2) is valid for any rheological model of a body.

Some essential remarks about the quantity γ should be made.

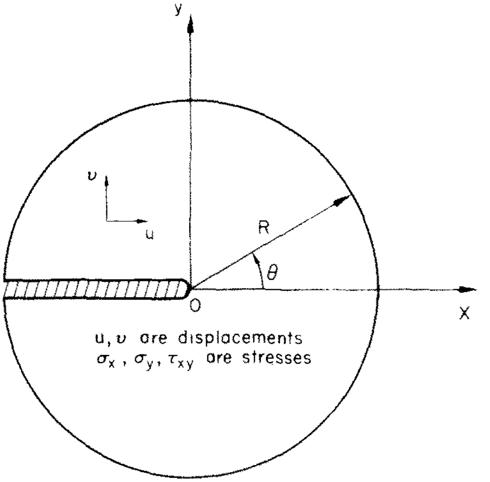

FIG. 1.

(a) The reversible part of the true surface energy γ is not equal to the total work of interatomic cohesion forces; it is equal only to some part of the work which is shaded in Fig. 2. This becomes apparent if we take into account the stability considerations during the process of moving apart two atomic planes. The rest of the work turns into sound and electromagnetic energy; this corresponds to the dynamic process. The radiation rate is directly proportional to the area of the new produced crack surface and is inversely proportional to the time of its production. These secondary radiation effects can be used, in principle, to monitor the fracture process (particularly, using the electromagnetic radiation emitted by the high power explosions in rocks, or the sound pulses produced by a separate growing crack).

(b) The quantity γ represents a physical constant of the material which depends only on the temperature T with the given environment. If we consider the relationship of the cohesion force versus the interatomic distance to be linear in the range of interest, then it is easily to obtain the following formula

$$\gamma = \frac{\delta^2 E}{a}. \tag{1.3}$$

Here E is Young's modulus, a is the interatomic distance in the absence of external forces $(\delta + a)$ is the interatomic distance corresponds to the maximum of cohesion force.

The value δ/a can be estimated physically as

$$\frac{\delta}{a} \approx \int_T^{T_f} \beta(T)\,\mathrm{d}T \tag{1.4}$$

Here $\beta(T)$ is the coefficient of thermal expansion, T_f is the fusion point. This estimate

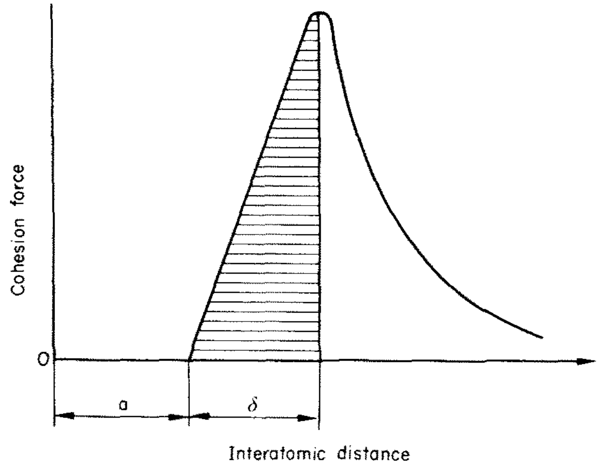

FIG. 2.

results from the fact, that the thermal expansion shifts to the right side of the curve in Fig. 2, so that the displacement δ corresponds roughly to the fusion point. The error present in equation (1.3) is of the order of magnitude of surface tension of the fused body. For most materials under normal conditions the value δ/a equals roughly 0·1.

The relationship of γ vs. T is accounted for by the thermal expansion and can be given in the following form

$$\gamma = \frac{E}{a}\left[\delta(0) - a\int_0^T \beta(T)\,\mathrm{d}T\right]^2 \tag{1.5}$$

In a rather broad temperature range it can be considered as a linear one

$$\gamma \approx \gamma(0) - 2\beta T E\delta(0).$$

B. *Elastic body*

In the dynamic case of a linearly elastic body the governing equation is written by means of equation (1.1) or (1.2) as

$$N^2 = \frac{4E\gamma R(m, \nu)}{\pi(1+\nu)m^2\sqrt{[1-m^2(1-2\nu)/(2-2\nu)]}}. \tag{1.6}$$

Here

$$R(m, \nu) = \sqrt{\left[(1-m^2)\left(1-\frac{1-2\nu}{2-2\nu}m^2\right)\right]} - (1-\tfrac{1}{2}m^2)^2,$$

$$m = \frac{l}{c_2}, \qquad c_2^2 = \frac{E}{2(1+\nu)\rho}, \qquad \nu \text{ is Poisson's ratio},$$

$$N = \lim_{y=0, x\to 0}[\sigma_y\sqrt{(x)}],$$

N is the stress intensity factor at the point O (Fig. 1). According to equation (1.6) the limiting crack velocity coincides with the Rayleigh velocity which is the root of the equation

$R(m_R, \nu) = 0$. The Mott's approximation which corresponds only to the first term of the expansion of the right hand part of equation (1.6) in the vicinity of zero turns out to be too rough. In homogeneous bodies the maximum velocity is limited even earlier by the crack-twinning velocity which is roughly equal to $0{\cdot}6\, C_2$.

In the quasistatic case for $m \to 0$ from equation (1.6) one can obtain the limiting Irwin's condition

$$N^2 = \frac{E\gamma}{\pi(1-\nu^2)}. \tag{1.7}$$

Owing to the local fracture character, the above equation can also be easily derived from the simple considerations of invariance. On the same grounds equation (1.7) is applicable also to the initial moment of the fracture of man-made cut bodies, the radius ρ_0 at the notch-root of which is much greater than the corresponding one for natural cracks; however, for notches the value γ_n depends essentially on the radius ρ_0

$$\gamma_n = \gamma f\left(\frac{\rho_0 \sigma_f^2}{E\gamma}\right). \tag{1.8}$$

Here σ_f is the fracture strength for the smooth specimen of radius ρ_0. In practice it is enough to retain the first two or three terms in the expansion of the function $f(x)$.

Equations (1.6) and (1.7) are applicable to glasses, ceramics, rocks, graphites etc.

C. *Viscoelastic body*

Equations (1.6) and (1.7) turn out to be valid also for the case of ideal linearly visco-elastic media, E and ν corresponding to the instantaneous deformation.

D. *Incompressible rigid–plastic body*

In such bodies the crack-growth proves to be impossible (the energy equation is not satisfied); and a crack enlarges as a cavity.

E. *Conclusions*

Let a body have any plastic properties which for large† deformations asymptotically approach the linear hardening ones, the viscous and creep properties being quite arbitrary. Consider the "superfine" structure, the characteristic linear dimension of which is small in comparison with the size of the plastic field in the vicinity of the crack end. In this most general case the governing equations (1.6) and (1.7) turn out to be valid within the framework of the small deformations theory for the growing crack tip. For this case N relates to the "superfine" structure, E and ν denote the corresponding constants for the asymptotically linear hardening range.

Notice that these conditions are fully determined by properties of the material just before the fracture occurs. Thus, according to equation (1.7), during loading a crack does not grow at first, until the equality (1.7) is attained. If the crack grows, equations (1.6) or (1.7) serve as the boundary conditions at the crack contour; in this case the main difficulties are presented in the solution of the mathematical problems concerning determination of the stress intensity factor of "superfine" structure. These mathematical difficulties

† As compared to limiting elastic deformations.

cause searching of other physical concepts which characterize the plastic field near the crack tip as a whole.

2. THE MODIFIED CONCEPT OF IRWIN–OROWAN

Irwin and Orowan introduced into fracture mechanics the idea that the dissipation energy magnitude per unit of the surface area of the quasistatic growing crack, γ_*, represents a material constant. Consider elastic–plastic media which asymptotically behave as ideal-plastic ones (the hardening prior to fracture is absent). In such a case, according to equation (1.7), the crack growth starts just after loading.

A. *Monotonic loading*

During monotonic loading the crack grows as well as the dimension of the plastic field near the crack tip until this dimension achieves an asymptotic value of the quasi-brittle fracture. The rigorous analysis results in the following governing equation which can be readily integrated, see Fig. 3

$$\frac{\mathrm{d}l}{\mathrm{d}N} = \frac{\alpha N^3}{K_c^2 - N^2}\left(K_c^2 = \frac{E\gamma_*}{\pi(1-v^2)}\right) \tag{2.1}$$

Here N is the stress intensity factor of fine structure, K_c is the Irwin's constant, α is a constant of the material. The value N determines the stress and strain distribution on distances large as compared to the dimension of the plastic field† and is calculated from the purely elastic problem.

B. *Cyclic loading*

The concepts mentioned above allow one to deduce the following formula for the crack growth velocity by cyclic loads

$$\frac{\mathrm{d}l}{\mathrm{d}n} = -\frac{1}{2}\alpha K_c^2\left(\frac{N_{\max}^2 - \delta N_{\min}^2}{K_c^2} + \ln\frac{K_c^2 - N_{\max}^2}{K_c^2 - \delta N_{\min}^2}\right) \tag{2.2}$$

$$(\delta = 1 \quad \text{for} \quad N_{\min} > 0, \qquad \delta = 0 \quad \text{for} \quad N_{\min} < 0)$$

Here n is the number of cycles, $N_{\max}$ and $N_{\min}$ are the maximum and minimum value of N during a cycle, respectively. For $N_{\max}/K_c \gtrsim 0{\cdot}5$, $N_{\min} = 0$ equation (2.2) coincides with the Paris formula $\mathrm{d}l/\mathrm{d}n \sim N_{\max}^4$.

The crack nonpropagation condition under cyclic loads has the form

$$N_{\max} < K_Y f(N_{\min}/N_{\max})(0 < K_Y < K_c, 0 < f < 1) \tag{2.3}$$

Here K_Y is a constant of the material analogous to K_c, f is a dimensionless function.

C. *Relation between γ and γ_*. Adsorbous effect*

The value γ_* represents a function of γ and other physical constants of the material because the approach treated in this section is to result in principle from the rigorous approach (Section 1). The energy considerations taking into account the finiteness of

† Small in comparison with the crack length.

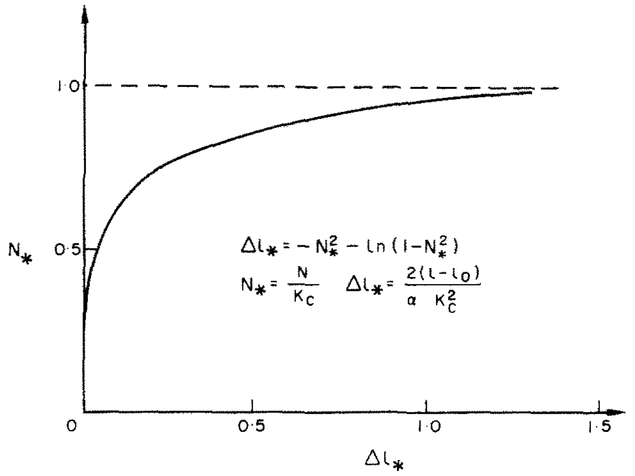

FIG. 3.

deformations near the crack tip lead to the following estimate: γ_*/γ is of the order of magnitute E/σ_s, where σ_s is the tension yield point of sufficiently thin patterns. The estimate allows one to elucidate the adsorbous effect of surface-active media on the material strength.

D. *Relation between* γ_* *and* T. *Cold brittleness*

The value γ_* grows with temperature T roughly proportionally to T; for metals there is a narrow range of critical temperatures $T_{min} \gtrsim T \gtrsim T_{min}+\Delta T$ where γ_* rises sharply from γ_{*min} to γ_{*max}. Within the temperature range the width of which is usually about 10–30°C the temperature relationship is well approximated by the formula

$$\gamma_* - \gamma_{*min} = (\gamma_{*max} - \gamma_{*min}) \exp\left[-\frac{\Delta T}{5(T - T_{min})}\right].$$

E. *Plates of finite thickness. Three constants theory*

In the case of through cracks in plates the value γ_* essentially depends on the plate thickness h because the mechanics of plastic yielding near the free surface of the plate, where plane stress takes place, is generally different from the yielding mechanics deep within the plate where plane strain takes place. This relationship can be represented as:

$$\gamma_* = \begin{cases} \gamma_{*II} & \text{for } h < h_* \\ \gamma_{*I} + (\gamma_{*II} - \gamma_{*I})\dfrac{h_*}{h} & \text{for } h > h_* \end{cases} \tag{2.4}$$

In practice for $h \sim h_*$ one observes a maximum of γ_* which is accounted for by free surfaces which mutually affect each other (dotted line in Fig. 4).

The values γ_{*I}, γ_{*II}, h_* represent material constants not depending on the plate thickness. The constant h_* is of special interest because plates of thickness h_* have a maximum specific strength. For several metals γ_{*II} is about ten times larger than γ_{*I}.

It is convenient to determine γ_* from experimental curves $\Delta l = \Delta l(N)$ with the help of formula (2.1) (see Fig. 3), or, making use of the physical meaning of γ_*, directly from the hysteresis loop "loading–unloading".

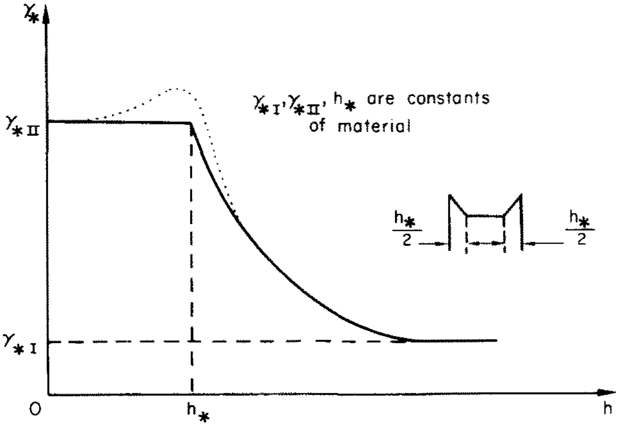

FIG. 4.

F. *Stability*

Let the function $N = N(p, l)$ be calculated from elastic stress analysis, where p is an outer load parameter and l is a crack length parameter being a function of p. The parametric stability condition of the crack growth can be reduced to the form

$$\left(\frac{K_c^2 - N^2}{\alpha N^3} - \frac{\partial N}{\partial l}\right) \Bigg/ \left(\frac{\partial N}{\partial p}\right) > 0 \tag{2.5}$$

The instability condition is obtained by changing the sign in (2.5). In particular, for quasi-brittle fracture (for $N = K_c$) from (2.5) one derives the stability condition

$$(\partial N/\partial l)/(\partial N/\partial p) < 0.$$

It is seen that the magnitude of the stress intensity factor for which the crack growth in an elastic–plastic body becomes unstable depends on the geometry of the body (e.g. on the crack length). This magnitude represents a constant of the material only for quasi-brittle fracture which is the case for any material for sufficiently large dimensions of a body and of a crack as it follows from the developed concepts.

Therefore, the approach which is based only on the stress intensity factor considerations and which is under development in the USA (ASTM theory) has limited possibilities. Also the "fine structure" and "stress intensity factor" concepts often lose their sense for real objects. The energy approach is more powerful; e.g. it allowed us also to solve the problems of an isolated crack in an infinite plate and in a strip for large dimensions of plastic zones.

G. *Size effect. Part through cracks*

The theory of size effect in the case of through cracks was briefly treated in Section 2.E. If a surface crack does not spread over the whole thickness of a plate, then the brittleness number of the plate is the following dimensionless parameter

$$\chi = \frac{E\gamma_*}{\sigma_s^2 L} \tag{2.6}$$

It shows also the safety factor of the plate service. Here L is the distance from the crack end to the opposite side of the plate.

For $\chi \sim 1$ or $\chi \gg 1$ a plate behaves as a totally plastic and safe one; the limiting load is independent of the crack size and is fully determined by the working cross-section of the plate (size effect is absent).

For $\chi \ll 1$ a plate behaves as a brittle one: the mean maximum stress calculated for the cross-section is dependent essentially on the crack size (or, for L/h = const, on L); there is size effect which can be easily estimated within the framework of the above concepts.

H. *Conclusions*

The Irwin–Orowan modified concept allows a satisfactory description of the main features of the crack growth in metals and polymers, provided that the times of loading are not too long (being different for different materials and temperatures). The values γ_* and α also depend, in general, on the deformation history (this includes the magnitude and the sign of preliminary plastic deformation causing blunting or sharpening of the crack, the number of loading cycles causing the material aging, etc.) and on the loading rate dN/dt (accounted for by the delay of plasticity). These secondary effects as well as the dynamics of the crack growth are not considered here for lack of space.

3. TIME EFFECTS

Let the crack growth result from the rupture at the crack tip owing to thermal fluctuations which form the basis of all irreversible time-dependent deformation processes. The rupture field near the crack tip increases in size during loading, even for N = const, until it attains an asymptotic value corresponding to the stationary regime of the crack growth. Consider the simplest and the most important time effects in quasistatic crack growth, when the Irwin–Orowan modified concept is not applicable.

A. *Stationary crack growth*

Using the concepts of fluctuation theory and of fine structure of the crack tip it is easy to obtain the following governing equation for the stationary regime

$$\frac{\partial l}{dt} = v \exp\frac{\zeta N - U}{kT}. \tag{3.1}$$

Here T is the absolute temperature, t is the time, k is the Boltzmann constant, v, ζ, U are constants of the material.

B. *Local aging*

In another limiting case, when the crack growth under constant external loads can be neglected, nevertheless, it may be necessary to take into account the processes of local aging at the crack tip. For constant N the fluctuation considerations lead to the following expression for the crack growth delay time

$$\tau = \tau_0 \exp\frac{U - \eta N}{kT} \qquad (\tau_0 = 10^{-12} \div 10^{-13}\,\text{sec}) \tag{3.2}$$

where τ_0, η, U are constants of the material.

In the case of N varying with time it is natural to apply the cumulative failure law

$$\int_0^\tau \exp\frac{\eta N(t)}{kT}\,dt = \tau_0 \exp\frac{U}{kT}. \tag{3.3}$$

The relationship (3.3) describes the incubation period of the crack growth.

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Абстракт—В настоящей работе приводится резюме последних исследований автора в области распространения трещина.